(14) 1. a) Suppose $f(x) = \dfrac{1}{3x+5}$. Use the **definition of derivative** to find $f'(x)$.

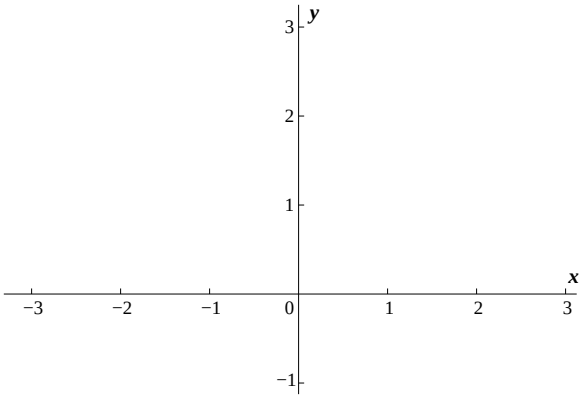
b) Use your answer to a) to write an equation for the line tangent to $y = \dfrac{1}{3x+5}$ when $x = -1$.

(14) 2. Suppose that the function $f(x)$ is described by

$$f(x) = \begin{cases} 2x+7 & \text{if } x < -2 \\ Ax^2 + B & \text{if } -2 \le x \le 1 \\ 2 - x & \text{if } 1 < x \end{cases}.$$

a) Find $A$ and $B$ so that $f(x)$ is continuous for all numbers. Briefly explain your answer.

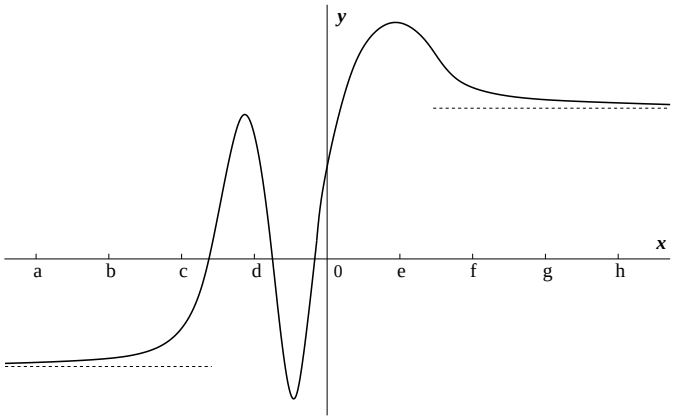
b) Sketch $y = f(x)$ on the axes given for the values of $A$ and $B$ found in a) when $x$ is in the interval $[-3, 3]$.



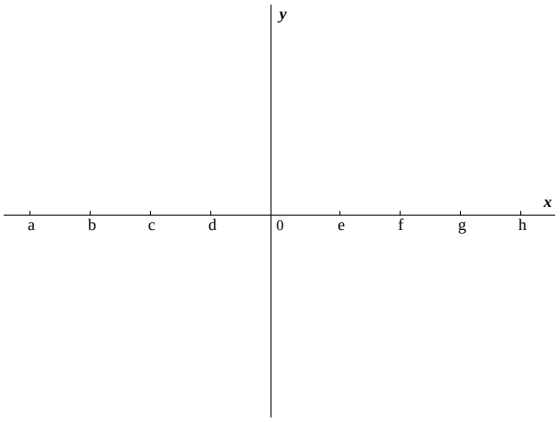

(20) 3. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x\to 1} \dfrac{x^2+4x-5}{x-1}$ b) $\lim\limits_{x\to 0} \dfrac{x^2}{\sin x}$ c) $\lim\limits_{x\to 3} \dfrac{\sqrt{x}-\sqrt{3}}{x-3}$ d) $\lim\limits_{x\to 1} \dfrac{3x-2}{\cos(\pi x)}$

(12) 4. Below is a graph of $y = f(x)$.




The graph of $y = f(x)$




The graph of $y = f'(x)$

Sketch a graph of $y = f'(x)$, the derivative of $f(x)$, on the axes given [below].

(8) 5. a) Find numbers $K$ and $L$ so that $K \le 2 + 5\sin\left(x^2+4x\right) \le L$ for all numbers $x$. Give evidence to support your answer.

b) Suppose $f(x) = x^3 + 2 + 5\sin\left(x^2+4x\right)$. What is the sign of $f(2)$? What is the sign of $f(-2)$? You may use your answers to a) to explain your answers here.

c) Explain why the equation $f(x) = x^3 + 2 + 5\sin\left(x^2+4x\right) = 0$ must have at least one solution. Give an interval in which a solution must be found. You must quote a specific result from this course and explain its relevance. Your answers to b) may be useful here.

(10) 6. a) If $f(x) = \dfrac{e^{2x}}{x^3 - 7}$, what is $f'(x)$? Please do not "simplify" your answer.

b) If $f(x) = (x^7 + \cos x)(4x^4 + 3x^3)$, what is $f'(x)$? Please do not "simplify" your answer.

c) Suppose that $f(x)$ is a differentiable function and that $f(1) = -2$, $f'(1) = 6$, $f(2) = 8$, and $f'(2) = 7$. Suppose that $g(x)$ is a differentiable function and that $g(1) = 2$, $g'(1) = 4$, $g(2) = 5$, and $g'(2) = -3$.
Suppose $F(x) = x^5 f(x)$. Compute $F'(1)$.

$F'(1) =$_____

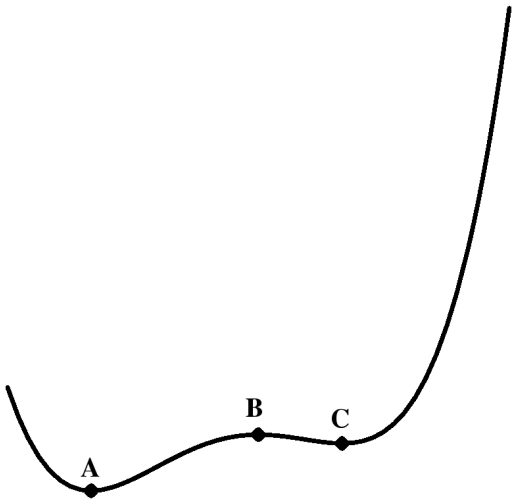
Suppose $G(x) = f(g(x))$. Compute $G'(1)$.

$G'(1) =$_____

(12) 7. The function $f(x)$ is defined by the formula

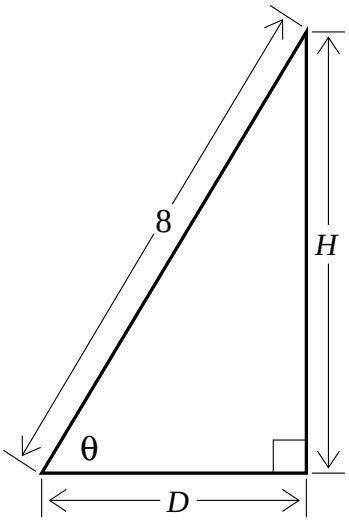
$$f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + 1$$

and a graph of the function, without any quantitative information or axes included, is below to the right. Use calculus to find exact coordinates of the points **A** and **B** and **C**.




Please write your answers here. **A** is (  ,  ). **B** is (  ,  ). **C** is (  ,  ).

(10) 8. A ladder which is 8 feet long has one end on flat ground and the other end on the vertical wall of a building. $H$ is the height from the ground to the point at which the ladder touches the building, and $D$ is the distance between the bottom of the ladder and the bottom of the wall. $\theta$ is the acute angle between the ladder and the ground.




a) Write $H$ as a function of $D$. That is, give a formula for $H$ involving $D$ and no other variable. What is the domain of this function when used to describe this problem? (The answer should be related to the problem's geometry.)

b) Write $H$ as a function of $\theta$. That is, give a formula for $H$ involving $\theta$ and no other variable. What is the domain of this function when used to describe this problem? (The answer should be related to the problem's geometry.)

# First Exam for Math 135, section F2

July 13, 2006

**NAME** _____

**Do all problems, in any order.**
**Show your work. An answer alone may not receive full credit.**
**No notes other than the distributed formula sheet may be used on this exam.**
**No calculators may be used on this exam.**

| Problem Number | Possible Points | Points Earned: |
|---|---|---|
| 1 | 14 | |
| 2 | 14 | |
| 3 | 20 | |
| 4 | 12 | |
| 5 | 8 | |
| 6 | 10 | |
| 7 | 12 | |
| 8 | 10 | |
| Total Points Earned: | | |