Formula sheet for the final exam in Math 135:F2, summer 2006

f is **continuous** at w if $\lim_{x \to w} f(x)$ exists and equals $f(w)$.

Intermediate Value Theorem
If $f(x)$ is continuous in $a \le x \le b$, then $f(x)$'s values include all numbers between $f(a)$ and $f(b)$.

$f'(x)$, the **derivative of f at** x, is defined to be $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

Mean Value Theorem
If $f(x)$ is differentiable in $a \le x \le b$, then there is at least one c between a and b so that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

A **Riemann sum** for $f(x)$ on $[a,b]$ is $\sum_{i=1}^{n} f(x_i^*)(x_i - x_{i-1})$. As the length of the subintervals $\to 0$, these sums $\to \int_a^b f(x)\,dx$, the **definite integral** of $f(x)$ on the interval $[a,b]$.

Fundamental Theorem of Calculus
- If $f(x)$ is continuous, then $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$.
- $\int_a^b f(x)\,dx = F(b) - F(a)$ if $F'(x) = f(x)$.

Function	Antiderivative
0	C (a constant)
x^n	$\frac{1}{n+1}x^{n+1} + C$ if $n \ne -1$
$x^{-1} = \frac{1}{x}$	$\ln(x) + C$
$kf(x)$ if k is a constant	$kF(x) + C$ if $F(x)$ is an antiderivative of $f(x)$
$f(x) + g(x)$	$F(x) + G(x) + C$ if $F(x)$, resp. $G(x)$, is an antiderivative of $f(x)$, resp. $g(x)$.
e^x	$e^x + C$
a^x	$\frac{1}{\ln(a)}a^x + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\tan x$	$\ln(\sec x) + C$

Substitution If $u = u(x)$, then $\int f(u(x))u'(x)\,dx = \int f(u)\,du$.

Function	Derivative
k (const.)	0
x^n	nx^{n-1}
e^x	e^x
$\ln x$	$1/x$
a^x	$a^x \ln(a)$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$(\sec x)^2$

Function	Derivative
$Cf(x)$	$Cf'(x)$ (C const.)
$f(x) + g(x)$	$f'(x) + g'(x)$
$f(x) \cdot g(x)$	$f'(x) \cdot g(x) + f(x) \cdot g'(x)$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - g'(x)f(x)}{g(x)^2}$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$

Area and Volume Formulas
Rectangle $A =$ LENGTH·WIDTH
Circle $A = \pi$ RADIUS^2
Triangle $A = \frac{1}{2}$ BASE·HEIGHT
Box $V =$ LENGTH·WIDTH·HEIGHT
Cylinder $V = \pi$ RADIUS^2·HEIGHT
Sphere $V = \frac{4}{3}\pi$ RADIUS^3

Exponential properties
$a^{b+c} = a^b \cdot a^c$ $\quad a^{-b} = 1/a^b$
$\left(a^b\right)^c = a^{bc}$ $\quad e \approx 2.718$
$a^0 = 1$ $\quad e^{\ln a} = a$ if $a > 0$

Logarithm properties
$\ln(a \cdot b) = \ln a + \ln b$ $\quad \ln(a^b) = b\ln(a)$
$\ln(a/b) = \ln(a) - \ln(b)$ $\quad \ln\left(\frac{1}{b}\right) = -\ln(b)$
$\ln(e^a) = a$ $\quad \ln(1) = 0$ $\quad \ln(e) = 1$

Special values of trig functions

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	1	0	Undefined

Miscellaneous formulas

If $a \ne 0$, the **roots** of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Distance between (a,b) and (c,d): $\sqrt{(a-c)^2 + (b-d)^2}$.

Circle center (h,k) and radius r: $(x-h)^2 + (y-k)^2 = r^2$.

Possibly useful numbers for this exam

$2^2 = 4$; $2^3 = 8$; $2^4 = 4^2 = 16$; $2^5 = 32$; $2^6 = 4^3 = 64$; $2^7 = 128$; $2^8 = 4^4 = 256$; $2^9 = 512$; $2^{10} = 4^5 = 1024$.
$3^2 = 9$; $3^3 = 27$; $3^4 = 81$; $3^5 = 243$.