7/31/2006

Math 135, section F2, summer 2006

Problems on implicit differentiation and linear approximation

Homework due on Tuesday, August 1

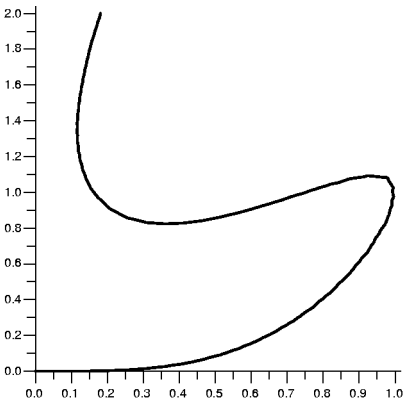
1. Suppose y is implicitly defined by x in the equation $x^2y - 5x = 2 - 3y$.

a) Solve for y as a function of x. Then find $\frac{dy}{dx}$.

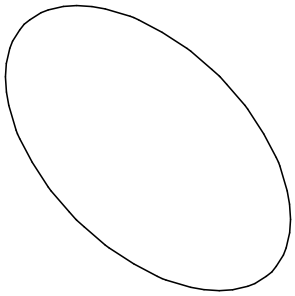
b) Find $\frac{dy}{dx}$ implicitly using the original equation.

c) Are the answers equal?

2. Part of the graph of $e^{y-xy} + 3\cos(x^2 - y) = 4$ is shown to the right. Check that the point $(1, 1)$ is on this curve. Suppose that y is implicitly defined by x near the point $(1, 1)$ by the equation given. Find the equation of the line tangent to the curve at $(1, 1)$. Does your answer "look" right?



3. To the right is a graph of $x^2 + xy + y^2 = 5$, a tilted ellipse. Find the equations of the *bounding box*. In computer graphics, this box is the smallest box with sides parallel to the vertical and horizontal axes (the x- and y-axes) which just contains the ellipse.



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4. Suppose that $f(x) = \sqrt[3]{6000x^2 - 5000}$. Please do *not* use a calculator in this problem.

a) Compute $f(1)$ and $f'(1)$.

b) Use linear approximation to approximate $f(1.04)$.

c) Use linear approximation to approximate $f(.97)$.

d) Compute $f''(1)$. Use this to answer the question(s): are the linear approximation values obtained in b) and c) likely to be larger than or smaller than the true values of $f(1.04)$ and $f(.97)$?

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5. Suppose a cube has edge length equal to E.

a) Find a formula for the volume, V, of the cube in terms of E. Find a formula for the total surface area, A, of the cube in terms of E. Find a formula for the total surface area, A, of the cube in terms of the volume, V.

b) Use linear approximation to find the approximate change in V as a function of E if E increases by 1.
Use linear approximation to find the approximate change in A as a function of E if E increases by 1.
Use linear approximation to find the approximate change in A as a function of V if V increases by 1.

c) Are these overestimates or underestimates of the true changes? *Explain* your answers! ("Pure thought" [and the second derivative] should be used here.)

6. The high-salaried MBA's and financial math wizards employed by the Amalgamated Widget Corporation have stated that when 3,000 widgets are sold, the profit is \$5,000,000, and the marginal profit (the hypothetical approximate result of selling one more widget), is \$700. One member of the sales staff states that, "A swing through the western states will accumulate orders for eight more widgets!" The travel costs for this trip are \$2,000. Is it worth it?* The people in marketing research believe that reducing advertising by \$10,000 has a 50% chance of shrinking widget sales by 20 widgets. Is it worth it?*

7. The classical (1916) DuBois and Dubois formula for the surface area of an adult human being is BODY SURFACE AREA $= (0.20247)W^{0.425}H^{0.725}$ where W is in kilograms and H is in centimeters. The result is given in meters^2. The currently recommended** Mosteller formula (reported in 1987) is BODY SURFACE AREA $= \sqrt{\frac{HW}{3600}}$.

At about 183 pounds and 73.5 inches, my body surface area is estimated as 2.07 m^2 (Mosteller) and 2.08 m^2 (Dubois).

I couldn't think of a neat problem with these formulas, but I wanted to show them to you.

Some numbers for problem 4: a) 10 and 40; b) 11.6; c) 8.8; d) –280 (approximate "true" values for b) and c) are 11.4206 and 8.6419); use concavity for your reasoning!

* And what is "it"?

** `http://www.halls.md/body-surface-area/refs.htm`