Math 135, section F2, summer 2006
Review problems for the first [semi] exam

Even compared to practices in our math summer courses, this exam is early. It will be about half as long as a standard first exam. The major purpose of the exam is to give early feedback to students about how they are doing in the course. The formally graded exam is the feedback method. This is especially valid since the overwhelming determinant of the course grade will be exam performance.

There are (many) more problems below than will be on the exam. The exam will cover the material of the first four course meetings. Problem solutions given by student volunteers will be discussed on Monday before the exam. I expect that most of the problems on the "real" exam will resemble problems given below. I hope you won't need a calculator for these problems. No calculator may be used on the exam.

1. Sketch the graph of a function $f(x)$ which has all of the following properties:

a) The domain of $f(x)$ is exactly $2 < x < 6$. All of the values of $f(x)$ are between 1 and 4.

b) $\lim_{x \to 6^-} f(x) = 3$ and $\lim_{x \to 2^+} f(x) = 2$.

c) $\lim_{x \to 4} f(x) = 2$ and $f(4) = 3$.

d) $\lim_{x \to 3^-} f(x) = 2$ and $\lim_{x \to 3^+} f(x) = 3$ and $f(3) = 2$.

e) If w is any number between 2 and 6 different from 3 and 4, then $\lim_{x \to w} f(x)$ exists and is equal to $f(w)$.

Note The preceding information certainly doesn't specify a unique function. You are asked to display the graph of one function satisfying all of the properties.

2. If $f(x) = \frac{1}{x^2}$, use algebraic properties of limits to compute $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

3. a) The limit $\lim_{x \to 3} \frac{x^2-9}{x-3}$ exists. What is it? Explain using algebra.

b) The limit $\lim_{x \to 2} \frac{x^2-2x}{\sqrt{x}-\sqrt{2}}$ exists. What is it? Explain using algebra.

4. It turns out that if $f(x) = \sqrt{x^3+1}$, then $f'(x) = \frac{3x^2}{2\sqrt{x^3+1}}$. You don't need to verify this assertion, but you may use it to answer the following question:

What is the equation of a line tangent to $y = \sqrt{x^3+1}$ when $x = 2$?

5. What is the domain of $f(x) = \frac{1}{(x-2)\sqrt{x-1}}$?

6. The point **A** has coordinates $(0, 2)$, the point **B**, $(3, 0)$, the point **C**, $(1, -1)$, and the point **D**, $(3, 1)$ as shown to the right. Use algebra to find the coordinates of the point where the lines **AB** and **CD** intersect.
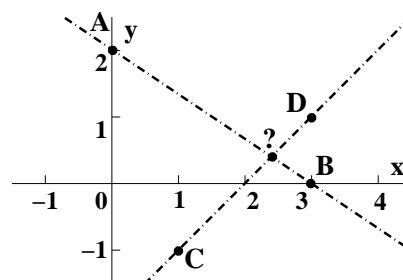
7. The graph of $f(x)$ is displayed to the right, using standard symbols to indicate where function values are and where they are not.
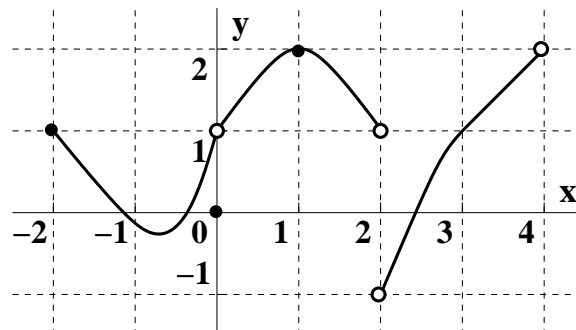
a) What, exactly, is the domain of $f(x)$? What, exactly, is the range of $f(x)$?

b) Create a table whose <u>column</u> labels are a, $f(a)$, $\lim\limits_{x \to a} f(x)$, $\lim\limits_{x \to a^-} f(x)$, and $\lim\limits_{x \to a^+} f(x)$. The

The first entries in each <u>row</u> of this table (that's the a column) should be -2, 0, 1, 2, and 4. Fill in all entries of the table. These entries should either be a specific real number which is the function value or limit value, or **DNE** if the requested entry does not exist.

8. θ is an unspecified angle in the diagram shown. Five of the other angles, as indicated, are right angles. Find the area of the shaded rectangle in terms of trig functions evaluated at θ and the two numbers given (4 and 7).
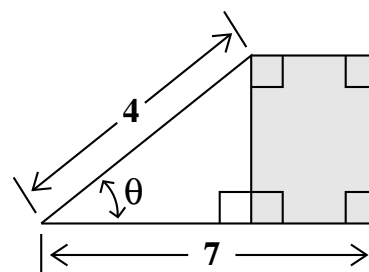
Note A helpful beginning step is to label every vertex in the picture!

9. Assume the following facts for this problem:

$$\lim_{x \to 5} f(x) = 3; \quad \lim_{x \to 5} g(x) = -7; \quad h(x) \text{ has domain } \mathbb{R} \text{ and } |h(x)| < 304 \text{ for all } x.$$

Either find the limits indicated below or state that they do not exist or declare that you can't decide if they exist from the information given. Support your answers *briefly* with some explanation or example as needed.

a) $\lim\limits_{x \to 5} 2f(x) + 9g(x)$

b) $\lim\limits_{x \to 5} \dfrac{f(x)}{g(x) - 2f(x)}$

c) $\lim\limits_{x \to 5} h(x)$

d) $\lim\limits_{x \to 5} 6 + (x - 5)h(x)$

10. a) What is $\lim\limits_{x \to 2\pi} \dfrac{\sin x}{x}$?

b) What is $\lim\limits_{x \to 2\pi} \dfrac{\sin x}{(x - 2\pi)}$?

c) What is $\lim\limits_{x \to 0} \dfrac{\sin x}{4x}$?

d) What is $\lim\limits_{x \to 2} (x - 2) \sin\left(\dfrac{5}{x^2} - \dfrac{2x+3}{x^4} + \dfrac{8}{x^7}\right)$?