6/28/2006

Math 135, section F2, summer 2006

An example of a function

Students sometimes object to examples which are given while discussing limits of functions. I thought I would provide an example of a naturally occurring* function to think about *before* complaining about the artificial complexity of any examples the instructor may give in this course. This function is defined *piecewise* in the following way.

The domain of the function $f(x)$ is all non-negative x: that is, $x \geq 0$. Here is how it is defined on its domain:

If $0 \leq x \leq 15{,}100$ then $f(x) = .1x$.

If $15{,}100 < x \leq 61{,}300$ then $f(x) = 1{,}510 + .15(x - 15{,}100)$.

If $61{,}300 < x \leq 123{,}700$ then $f(x) = 8{,}440 + .25(x - 61{,}300)$.

If $123{,}700 < x \leq 188{,}450$ then $f(x) = 24{,}040 + .28(x - 123{,}700)$.

If $188{,}450 < x \leq 336{,}550$ then $f(x) = 42{,}170 + .33(x - 188{,}450)$.

If $336{,}550 < x$ then $f(x) = 91{,}043 + .35(x - 336{,}550)$.

There is standard math notation for piecewise defined functions. Here is $f(x)$ described using this notation.

$$f(x) = \begin{cases} .1x & \text{if } 0 \leq x \leq 15100 \\ 1510 + .15(x - 15100) & \text{if } 15100 < x \leq 61300 \\ 8440 + .25(x - 61300) & \text{if } 61300 < x \leq 123700 \\ 240410 + .28(x - 123700) & \text{if } 123700 < x \leq 188450 \\ 42170 + .33(x - 188450) & \text{if } 188450 < x \leq 336550 \\ 91043 + .35(x - 336550) & \text{if } 336550 < x \end{cases}.$$

What the *heck* is this?

Such a function is called a *piecewise linear* function: its graph is made of segments of straight lines.

Where do all of the peculiar numbers come from? Why are they specified so precisely? Who could care? Do these numbers have some strange significance?

---

* **Joke**, but this *is* a real-world function that many people care about.