5. a) Suppose that f is a real-valued C^1 function on the open unit interval, $(0, 1)$, of $\mathbb{R}$ and that f' is never zero. Prove that f is 1-to-1.

b) Suppose that f is a complex analytic function defined on the open unit disc, $\{z \in \mathbb{C} : |z| < 1\}$ and that f' is never zero. Give an example to show that such a function need *not* be 1-to-1.

6. a) Suppose that f is holomorphic and nowhere 0 on a bounded region of the plane. If f extends continuously to the closure of the region and has constant modulus on the closure of the region, prove that f is constant.

b) Can the hypothesis that f is "nowhere 0" be omitted with the conclusion still valid?

c) Can the hypothesis "on a bounded region of the plane" be replaced by "on a region in the plane whose complement is unbounded" with the conclusion still valid?

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