1. a) Suppose f is an entire function, and suppose that f is also in $L^2(\mathbb{R}^2)$. Prove that f is the zero function.

b) Give an example of a non-zero C^∞ function in $L^2(\mathbb{R}^2)$.

2. Consider the infinite series

$$\sum_{n=1}^{\infty} \frac{z^n}{2+z^n}$$

Warning: this is not a power series! Therefore any results you know about power series are not directly applicable.

a) Find the largest open subset of $\mathbb{C}$ for which this series converges.

b) Show that the series converges uniformly on the compact subsets of the open set identified in a).

c) The series represents a holomorphic function in the open set found in a). What is the derivative of this function at $z = 0$?

Source: H. P. Boas, Texas A&M