Problem statement A function $f(x, y)$ is called *homogeneous of degree k* if the equation $(*)$ given by $f(tx, ty) = t^k f(x, y)$ is true for all x, y, and t.

a) Suppose $f(x, y) = x^3y - 5x^2y^2$. Show that $f(x, y)$, $f_x(x, y)$ and $f_y(x, y)$ are each homogeneous. What are the degrees of homogeneity? Also verify that $xf_x(x, y) + yf_y(x, y) = 4f(x, y)$.

b) Suppose $f(x, y)$ is any function that is homogeneous of degree k. Show that $f_x(x, y)$ is homogeneous of degree $k - 1$.

Hint Apply $\partial/\partial x$ to each side of $(*)$ and use the Chain Rule.

c) Suppose $f(x, y)$ is any function that is homogeneous of degree k. Show that $xf_x(x, y) + yf_y(x, y) = kf(x, y)$.

Hint Apply $\partial/\partial t$ to each side of $(*)$ using the Chain Rule. Then set $t = 1$.