

Problem statement Suppose that $F(x, y)$ is a function all of whose second partial derivatives exist and are continuous. Suppose you also know:

$$F(0, 0) = a, \quad \frac{\partial F}{\partial x}(0, 0) = b, \quad \frac{\partial F}{\partial y}(0, 0) = c, \quad \frac{\partial^2 F}{\partial x^2}(0, 0) = d, \quad \frac{\partial^2 F}{\partial x \partial y}(0, 0) = e, \quad \frac{\partial^2 F}{\partial y^2}(0, 0) = f$$

and that $G(s, t) = F(3s + 2t, st)$.

a) Compute $\frac{\partial G}{\partial s}$ (your answer should be expressed in terms of s , t , and partial derivatives of F).

b) Use your answer to a) to compute $\left(\frac{\partial}{\partial t} \left(\frac{\partial G}{\partial s}\right)\right)(0, 0)$ in terms of a , b , c , d , e , and f .

c) Compute $\frac{\partial G}{\partial t}$.

d) Use your answer to c) to compute $\left(\frac{\partial}{\partial s} \left(\frac{\partial G}{\partial t}\right)\right)(0, 0)$ in terms of a , b , c , d , e , and f .

Do your answers to b) and d) satisfy Clairaut's Theorem?