Problem statement The 2×2 determinant can be thought of as a function which takes four variables as input, and returns a real number as output:

$$\det(a, b, c, d) = \det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

a) What is the gradient of this function, $\nabla \det$? (The gradient of any function is a vector. First question: how many components will $\nabla \det$ have?)

b) If $a = 2$, $b = -3$, $c = 4$, and $d = 5$, then

$$\det(a, b, c, d) = \det(2, -3, 4, 5) = \det \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix} = 22.$$

Suppose we want to change each of a, b, c, and d by a little bit, where "little bit" here means that $(\triangle a)^2 + (\triangle b)^2 + (\triangle c)^2 + (\triangle d)^2 \leq .01$. If we want to make changes so the new determinant is as *large* as possible, what changes would you recommend?