Problem statement If $u(x,t)$ is the temperature in a rod at time t a distance x from some fixed point, then to a good approximation $u(x,t)$ satisfies the *Heat Equation*:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial t^2}$$

where D is a constant which describes the physical properties of the rod. (To be more precise, the Heat Equation applies when the surface of the rod is well insulated, so that heat can enter or leave the rod only through its ends. The same equation also appears in the analysis of many diffusion problems.)

For $t > 0$, define $u(x,t) = \dfrac{e^{\left(\frac{-kx^2}{t}\right)}}{\sqrt{t}}$ where k is a constant. There is one value of k for which this function is a solution of the Heat Equation. Find the value of k and verify that the resulting function does solve the equation. The value of k will be related to D.

Comment This is the most famous solution of the Heat Equation and is called the *Fundamental Solution*.