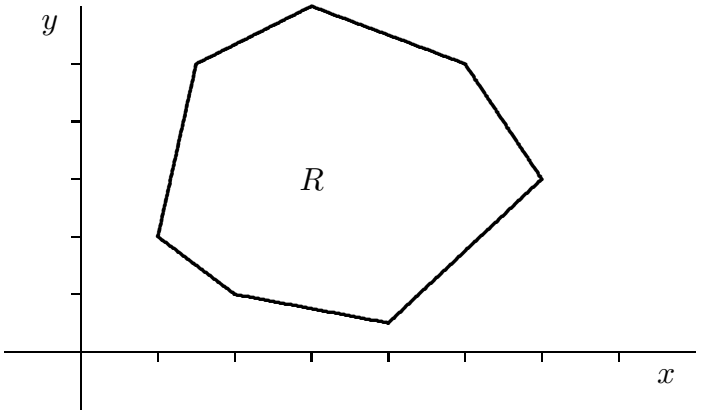
Problem statement A *linear programming* problem is a max-min problem of a somewhat different character from those we encounter in Math 251. In the simplest case of such a problem we want to maximize and/or minimize a linear function of x and y, that is, a function $f(x, y) = ax + by$, over a polygonal region like the region R below. (There are many real applications with problems of this sort, and many more than two variables may be involved – perhaps thousands.)



Suppose now that the function whose extrema we want to find is $f(x, y) = x + 2y$.

a) Find all critical points of f (pretty easy, yes?). What does this result tell you about where the extreme values of f in R occur?

b) Draw a region R that looks something like the above (the picture is not critical) and on the same figure sketch and label some level curves of the function f, including some that cross R. From your picture, explain why the maximum and minimum values of f in R will be taken on at vertices (corners0 of the region R, and how you would determine the relevant vertices graphically.

c) Suppose now that we consider a region R (not the one in the picture!) consisting of the points (x, y) which satisfy all of these inequalities:

$$y \geq 3 - x, \quad 2y \geq x, \quad y \leq 3x - 1, \quad y \geq 2x - 6, \quad 3y \leq 17 - x.$$

Make a *careful* sketch of R, finding the coordinates of all the vertices. By adding some level curves of f, determine at which vertices of R the maximum and minimum of f occur, and from this find these extreme values. Check your analysis by evaluating f at all the vertices.