

Problem statement Suppose that \mathbf{a} and \mathbf{b} are two three-dimensional vectors.

a) Use the identity $(\cos \theta)^2 + (\sin \theta)^2 = 1$ to derive an identity involving $\mathbf{a} \cdot \mathbf{b}$, $\mathbf{a} \times \mathbf{b}$, $\|\mathbf{a}\|$, and $\|\mathbf{b}\|$.

b) Verify this identity algebraically using the formulas

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3; \quad \mathbf{a} \times \mathbf{b} = \det \begin{pmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}.$$