Problem statement A thin plate (lamina) of uniform density ρ covers the portion of the xy plane lying in the first quadrant and between two circles centered at the origin. The inner circle has radius a and the outer, b. The coordinates of points covered by this plate satisfy $x, y \ge 0$ and $a^2 \le x^2 + y^2 \le b^2$ as shown.

a) Find the coordinates (\bar{x}, \bar{y}) of the center of mass P of this plate. (How can symmetry simplify this problem?)

b) Suppose now that b = 1. From your answer in a) determine for what values of a the center of mass P lies inside the area covered by the plate (as shown in the figure).

