Problem statement Suppose that C_1 and C_2 are two curves in the plane, given parametrically by

$$C_1\colon \begin{cases} x(t) = 0 \\ y(t) = 2t - 1 \end{cases} \text{ for } 0 \le t \le 1\,; \quad C_2\colon \begin{cases} x(t) = -\cos t \\ y(t) = \sin t \end{cases} \text{ for } -\tfrac{\pi}{2} \le t \le \tfrac{\pi}{2}\,.$$

a) Sketch these curves.

b) If f is a function defined in the plane, let $I_1(f)$ and $I_2(f)$ be the line integrals of f over these curves with respect to arc length: $I_1(f) = \displaystyle\int_{C_1} f(x,y)\,ds$ and $I_2(f) = \displaystyle\int_{C_2} f(x,y)\,ds$. In each case below determine which of $I_1(f)$ and $I_2(f)$ is greater or whether they are equal (that is, whether $I_1(f) > I_2(f)$, $I_1(f) < I_2(f)$ or $I_1(f) = I_2(f)$) *without evaluating the integrals*. Explain your reasoning carefully. Then check your answer by computing the integrals.

i) $f(x,y) = 17$; ii) $f(x,y) = x$; iii) $f(x,y) = y$.