Problem statement Let C_1 be the circle $x^2+y^2=16$ and C_2 the circle $(x-1)^2+y^2=4$, each oriented counterclockwise. Suppose $P(x,y)=\dfrac{x}{x^2+y^2}+y$ and $Q(x,y)=\dfrac{y}{x^2+y^2}-x$.

a) Verify that $\dfrac{\partial Q}{\partial x}(x,y)-\dfrac{\partial P}{\partial y}(x,y)=-2$ for $(x,y)\neq(0,0)$.

b) Explain why Green's Theorem cannot be used to calculate either $\displaystyle\int_{C_1}(P\,dx+Q\,dy)$ or $\displaystyle\int_{C_2}(P\,dx+Q\,dy)$.

c) Use Green's Theorem to show that $\displaystyle\int_{C_1}(P\,dx+Q\,dy)=\int_{C_2}(P\,dx+Q\,dy)-24\pi$.