Problem statement a) A certain function $f(x, y)$ is known to have partial derivatives of the form

$$\frac{\partial f}{\partial x} = 2xy + g(y), \qquad \frac{\partial f}{\partial y} = x^2 + 3x. \tag{*}$$

Note that g is a function of y only. Use the equality of mixed partial derivatives (Clairaut's Theorem) to find the function g up to an arbitrary additive constant. Then find all functions f with partial derivatives of the form $(*)$.

b) Find all functions $f(x, y)$ satisfying

$$\frac{\partial f}{\partial x} = 2e^{2x}y + xy^2 + g(y), \qquad \frac{\partial f}{\partial y} = x^2 y + 4y^3 x + h(x).$$

Hint The solution contains *four* arbitrary constants.