

Problem statement a) Suppose that $z = f(x, y)$, that $x = g(t)$ and $y = h(t)$, and that the functions f , g , and h are twice differentiable. Use the Chain Rule to find an expression for $\frac{d^2z}{dt^2}$.

b) An insect crawls on a metal plate in the plane. At time $t = 1$ its position vector is $\mathbf{i} + 2\mathbf{j}$, its velocity is $2\mathbf{i} - \mathbf{j}$, and its acceleration is $3\mathbf{i} + 4\mathbf{j}$. Suppose that the temperature of the plate at the point x, y is a certain function $T(x, y)$ satisfying

$$\begin{aligned} T(1, 2) &= 2, & T_x(1, 2) &= -1, & T_y(1, 2) &= 3, \\ T_{xx}(1, 2) &= 0, & T_{xy}(1, 2) &= 1, & T_{yy}(1, 2) &= -2. \end{aligned}$$

If $T(t)$ is the temperature experienced by the insect at time t , find $\frac{dT}{dt}$ and $\frac{d^2T}{dt^2}$ at time $t = 1$.