

Problem statement Suppose $f(x) = x^3 + x - 1$.

a) Explain why f has a root in the interval $[0, 1]$.

b) Suppose A is a constant and $g(x) = x^3 + x - 1 + Ax(x - 1)(2x - 1)$. Show that g has at least one root in the interval $[0, 1]$.

c) Calculate $g\left(\frac{1}{3}\right)$ and $g\left(\frac{2}{3}\right)$. If A is large enough, show that g must have three roots in the interval $[0, 1]$.