

Problem statement In this problem two different families of curves are considered. The families are said to be *orthogonal* to one another, since at every point where a curve from the first family intersects a curve from the second family, the tangent lines of the two curves are at right angles.

a) Sketch the curves $x^4 - 3y^4 = C$ for $C = -1, 0, 1,$ and 2 in the viewing window $[.5, 2] \times [.25, 1.25]$. Also, sketch the curves $3x^{-2} + y^{-2} = D$ for $D = 3, 4, 5,$ and 6 in the same window.

Suggestion: First solve to find y explicitly as a function of x , remembering that $y > 0$ in the region given.

b) Use calculus to verify that the two families are orthogonal.

Suggestion: Use implicit differentiation on the equation that defines each family of curves and express $\frac{dy}{dx}$ as a function of x and y .