Problem statement A continuous function f is defined on the interval $[-2, 2]$. The values of f at some of the points of the interval are given by the following table:

x	-2	-1	0	1	2
$f(x)$	2	-1	2	-1	2

a) Using only this information, what can be concluded about the roots of f, that is, the solutions of $f(x) = 0$, in the interval $[-2, 2]$? The answer should be something like: f has at least 8 roots in $[-2, 2]$, or f has at most 6 roots in $[-2, 2]$.

Suggestion Use the Intermediate Value Theorem on each of the intervals $[-2, -1]$, $[-1, 0]$, $[0, 1]$, and $[1, 2]$.

b) If $f(x) = x^4 - 4x^2 + 2$, verify that the relevant values of f are given by the table above.

i) Sketch the graph of $y = f(x)$ in the viewing window $[-2.5, 2.5] \times [-3, 3]$.

ii) How many roots does f have in the interval $[-2, 2]$? Find the roots algebraically. *Suggestion:* Let $t = x^2$ and solve with the quadratic formula. Then find x.

c) If $f(x) = x^4 - 4x^2 + 2 + 5(2x - 1)x(x^2 - 1)(x^2 - 4)$. Verify that the relevant values of f are given by the table above.

i) Sketch the graph of $y = f(x)$ in the viewing window $[-2.5, 2.5] \times [-80, 80]$.

ii) Explain why f has at least one root in each of the intervals $(-2, 1)$, $(-1, 0)$, $(0, 1)$, and $(1, 2)$.

iii) Sketch the graph of $y = f(x)$ in the viewing window $[0, 1] \times [-1, 3]$.

iv) How many roots does f have in the interval $[0, 1]$?

Approximate the roots of f in $[0, 1]$ to three decimal places using a calculator.

d) Having done b) and c), was your original conclusion in part a) correct?