Problem statement Solutions of this equation

$$\text{(HE)} \qquad \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

describe *steady state heat flow* in a thin plate. The partial differential equation is called the *Heat Equation.*

a) Verify that if $H(x,y) = 3x^2 - 3y^2 + 5xy + 4x - 2y + 7$, then this H is a solution of the equation (HE). What are all the critical points of this H? What type of critical point (max, min, saddle) is each of them?

b) Verify that if $H(x,y) = \cos x \sinh y$*, then this H is a solution of the equation (HE). What are all the critical points of this H? What type of critical point (max, min, saddle) is each of them?

c) Make a guess about the kind of critical point of solutions of the equation (HE) can have. Prove that **if** the second derivative test can be applied, then any critical point must be one of the type you asserted. Your assertion corresponds to a physically "reasonable" property of heat flow: it doesn't focus or lump up in the absence of heat sources.

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* The function "sinh y" (sinh is pronounced like "cinch") is the hyperbolic sine of y, and is defined by $\sinh y = \frac{e^y - e^{-y}}{2}$.