Problem statement Given n data points $(x_1, y_1), \ldots, (x_n, y_n)$, we may seek a linear function $y = mx + b$ that best fits the data. The **linear least-squares fit** is the linear function $f(x) = mx + b$ that minimizes the sum of the squares (see the Figure) $E(m, b) = \sum_{nj=1}^{n} (y_j - f(x_j))^2$.

Show that E is minimized for m and b satisfying

$$m \sum_{j=1}^{n} x_j + bn = \sum_{j=1}^{n} y_j \text{ and } m \sum_{j=1}^{n} x_j^2 + b \sum_{j=1}^{n} x_j = \sum_{j=1}^{n} x_j y_j$$

Comment This is problem 44 in the textbook's section 14.7. The result is quite important in practical computation. Several assertions must be verified: that E has one critical point which is a local minimum, and that this local minimum is actually an *absolute* minimum.