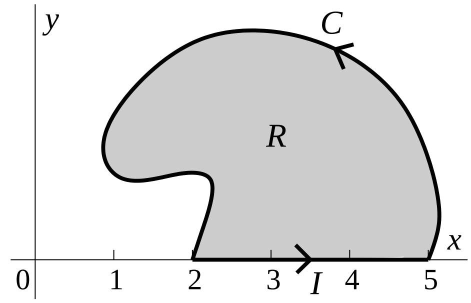
Problem statement A region R in $\mathbf{R}^2$ is located in the first quadrant, as shown. Its boundary, oriented counterclockwise as shown, is an interval $I = [2, 5]$ on the x-axis and a curve C in the first quadrant.



Suppose the following information is also known:

$$\iint_R 1\,dA = 5\,; \quad \iint_R x\,dA = 12\,; \quad \iint_R y\,dA = 8\,.$$

Find $\displaystyle\int_C \left(x^2 + xy + 3y\right) dx + \left(\arctan(y^3) + 3x^2 + 2xy + x\right) dy.$

Hint $I+C$ is a positively (counterclockwise) oriented piecewise smooth simple closed curve which is the boundary of R. Be careful because the formulas for $P(x, y) = x^2 + xy + 3y$ and $Q(x, y) = \arctan(y^3) + 3x^2 + 2xy + x$ together have seven "pieces".