Problem statement Polynomials have roots. For example, $(x-1)(x+2)(x-3) = x^3 - 2x^2 - 5x + 6$ which means that $x^3 - 2x^2 - 5x + 6$ is 0 when $x = 1$ or $x = -2$ or $x = 3$. Low degree polynomials have algebraic recipes for roots in terms of their coefficients. There are no such formulas for higher degree polynomials. If r_1, r_2, and r_3 are the roots of a third degree polynomial, $x^3 + Ax^2 + Bx + C$, then the coefficients (A, B, and C) are functions of the roots (r_1, r_2, and r_3).

a) What are the functions? That is, write A, B, and C as functions of r_1, r_2, and r_3. Verify if $\begin{cases} r_1 = 1 \\ r_2 = -2 \\ r_3 = 3 \end{cases}$ then $\begin{cases} A = -2 \\ B = -5 \\ C = 6 \end{cases}$.

b) Suppose the roots are changed: $\begin{cases} r_1 : 1 \to 1.02 \\ r_2 : -2 \to -2.04 \\ r_3 : 3 \to 2.95 \end{cases}$. Use partial derivatives and linearization to predict the *approximate* changes in the coefficients.

c) Suppose now that the coefficients are changed. That is, consider new coefficients: $\begin{cases} A = -2.03 \\ B = -5.02 \\ C = 6.01 \end{cases}$. Approximate the roots which would give these coefficients. (This is harder, and a *new idea* is needed: what perturbations in the roots will, to first order, give these perturbations in the coefficients? A "system" of three linear equations in three unknowns must be solved.)