

Problem statement The equation below is sometimes attributed to Atle Selberg, a prominent and long-lived Norwegian mathematician (1907–2007) with the remark that he discovered it while a teenager. This is a nice legend, but the problem apparently also appears in an English calculus book of the 1920's and perhaps before. So: verify that

$$\int_0^1 x^{-x} dx = \sum_{n=1}^{\infty} \frac{1}{n^n}.$$

Hint Get a reduction formula for $\int x^n (\ln x)^m dx$, and use it improperly. The value of both sides of the equation is approximately 1.291285997. I don't know any use for this formula besides its lovely existence.