Problem statement One of the physicists' favorite methods of computing definite integrals is illustrated by the examples in this problem. It is a *very* slick trick. The general idea is: put in a parameter, differentiate with respect to that parameter, and see what happens.

a) Suppose $F(a) = \int_0^\infty e^{-ax}\,dx$, where a is a positive number. Compute $F(a)$ directly. Then differentiate both sides N times with respect to a (the definite integral itself and its value!), and set $a = 1$. The results are integral formulas used in statistics (the Gamma (Γ) function) and in computing Laplace transforms (with many engineering applications).

b) Suppose $G(a) = \int_0^\infty \frac{1}{x^2 + a}\,dx$, where a is a positive number. Compute $G(a)$ directly. Then differentiate both sides (the definite integral itself and its value!) 3 times with respect to a. The result, after a little bit of algebra, will be a formula for $\int_0^\infty \frac{1}{(x^2 + a)^4}\,dx$. This formula can be obtained using partial fractions, but this method is much faster.

Comment A mathematician might say, "The justification of this method is not obvious." A physicist might reply, "But it works."