Problem statement Suppose $f(x)$ is a differentiable function with $f(-1) = 2$ and $f(2) = -1$. The differentiable function $g(x)$ is defined by the formula $g(x) = f(f(x))$.

a) Compute $g(-1)$ and $g(2)$. Explain why $g(x) = 0$ must have at least one solution A between -1 and 2.

b) Compute $g'(-1)$ and $g'(2)$ in terms of values of f and f'. Verify that $g'(-1) = g'(2)$. Explain why $g''(x) = 0$ must have at least one solution B between -1 and 2.

c) Suppose now that $f(x) = Cx^2 + D$. Find values of C and D so that $f(-1) = 2$ and $f(2) = -1$. Compute $g(x) = f(f(x))$ directly for those values of C and D, and use algebra on the resulting formulas for $g(x)$ and $g''(x)$ to find numbers A and B between -1 and 2 so that $g(A) = 0$ and $g''(B) = 0$. The "abstract" assertions of a) and b) should be verified.