

Problem statement Suppose $5x^3y - 3xy^2 + y^3 = 6$. $(1, 2)$ is a point on this curve. Is the curve concave up or concave down at $(1, 2)$?

Explicit way to go y can be solved as a function of x .^{*} Then you can differentiate the formula twice and evaluate when $x = 1$.

Implicit way to go Find $\frac{dy}{dx}$ implicitly and then differentiate again to get $\frac{d^2y}{dx^2}$. Evaluate everything at $(1, 2)$.

^{*} Here it is (really!):

$$y = \left(-\frac{5}{2}x^4 + 3 + x^3 + \frac{1}{18}\sqrt{1500x^9 - 675x^8 - 4860x^4 + 2916 + 1944x^3} \right)^{1/3} - \frac{\frac{5}{3}x^3 - x^2}{\left(-\frac{5}{2}x^4 + 3 + x^3 + \frac{1}{18}\sqrt{1500x^9 - 675x^8 - 4860x^4 + 2916 + 1944x^3} \right)^{1/3}} + x$$