Problem statement Define $S_6(n) = 1^6 + 2^6 + 3^6 + \cdots + n^6$ (in summation notation, $S_6(n) = \sum_{k=1}^{n} k^6$) then an explicit formula for $S_6(n)$ is known, and it is:

$$S_6(n) = \frac{1}{42}\left(6n^7 + 21n^6 + 21n^5 - 7n^3 + n\right) .$$

Similar formulas are known for other powers. These are sometimes called, collectively, *Faulhaber's formula*. Jacob Bernoulli also discovered these formulas but Faulhaber published earlier. It isn't even clear that the values of this formula are integers when n is an integer! **Assume that this formula is true.**

a) Check the formula for S_6 by evaluating it for $n = 4$. The answer should be the same as $1^6 + 2^6 + 3^6 + 4^6$.

b) Find some area and some approximating sum for this area which knowledge of this formula will allow you to evaluate exactly. Write the approximating sums, and evaluate the limit of these sums as $n \to \infty$ to compute the area.