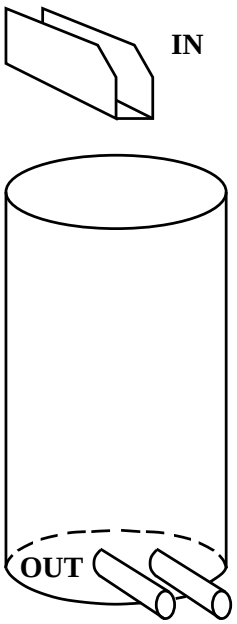
Problem statement A waste holding tank for an industrial process is constructed as shown to the right. The cross-sectional area of the holding tank, a cylinder, is 5 square feet, and the tank's height is 15 feet. Assume that the effluent is entering the top of the tank using the sluiceway shown and the rate of fluid entering the tank is modeled by the periodic function $f(t) = 3.5 + \sin\left(\frac{2\pi}{24}t\right)$. $f(t)$ is measured in cubic feet per hour and t is measured in hours, with $t = 0$ being midnight.



Each of the pipes which empty the tank has a carrying capacity of 2 cubic feet per hour. One pipe is always open. The other pipe is open from $t = 12$ (noon) until $t = 24$ (the next midnight). You may assume that when a pipe is open, its carrying capacity is fully used.

The tank at time $t = 0$ contains 10 cubic feet of fluid, so the fluid depth is 2 feet. What is the depth at the next midnight, when $t = 24$? Does the fluid overflow the tank during the first 24 hours, where $0 \leq t \leq 24$?

If this model is accurate, does the fluid ever overflow the tank?