Problem statement Consider line segments which are tangent to a point on the right half (x > 0) of the curve $y = x^2 + 1$ and connect the tangent point to the x-axis. Several are displayed in the diagram to the right. If the tangent point is close to the y-axis, the line segment is long. If the tangent point is far from the y-axis, the line segment is also very long. Which tangent point has the shortest line segment?



How to get started Suppose C is a *positive* number. What point on the curve has first coordinate equal to C? What is the slope of the tangent line at that point? Find the *x*-intercept of the resulting line. Compute the distance between the point on the curve and the *x*-intercept, and find the minimum of the *square* of that distance (minimizing the square of a positive quantity gets the same answer as minimizing the quantity, and here we get rid of a square root).