Problem statement A graph of the derivative of f(x) is displayed below. Information about the function f(x) is known only for -2.5 < x < 3.5. Also f(-2) = 1. Consider the graph carefully, and consider the information in both the numbers and the shapes of the graph (both "quantitative" and "qualitative" information)!



a) Explain why -2 < f(0) < -1. Look carefully at the graph and make estimates using the MVT. Explain the steps of your reasoning in detail.

b) Explain why f(3) > 4 + f(1). Again, use the MVT and explain your reasoning in detail.

c) How big and how small can f(1) - f(0) be?

d) Use the information in a), b), and c) to explain why f(3) must be positive.

e) Explain why f(x) = 0 must have a solution between 0 and 3. Use the IVT and the information obtained in previous parts of this problem.