Problem statement One of the Bessel functions used to describe the vibration of a circular plate is defined by this infinite series: $J(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$.

a) Show that this series converges absolutely for all values of x.

b) Explain briefly why the result of a) implies that the series converges for all x.

c) Here are individual terms of the series for two values of x and for some values of n.

$\frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$x=1$	1	$-\frac{1}{4}$	$\frac{1}{64}$	$-\frac{1}{2{,}304}$	$\frac{1}{147{,}456}$	$-\frac{1}{14{,}745{,}600}$
$x=4$	1	-4	4	$-\frac{16}{9}$	$\frac{4}{9}$	$-\frac{16}{225}$

Use entries of this table and facts about the series to explain why $J(1)$ must be positive and $J(4)$ must be negative.

Hint Select an N for each x and split the sum: $\sum_{n=0}^{\infty} = \sum_{n=0}^{N} + \sum_{n=N+1}^{\infty}$. Evaluate the finite sum explicitly and estimate the infinite tail $\sum_{n=N+1}^{\infty}$.