Problem statement A computer reports the following information:

$$\sum_{j=1}^{10} \frac{1}{j^3+2j^2+j} \approx 0.35105\,; \quad \sum_{j=1}^{100} \frac{1}{j^3+2j^2+j} \approx 0.35501\,; \quad \sum_{j=1}^{1000} \frac{1}{j^3+2j^2+j} \approx 0.35506\,.$$

This suggests that $\sum_{j=1}^{\infty} \frac{1}{j^3+2j^2+j}$ converges and that its sum is 0.355 (to 3 decimal places). Explain the details in the following outline of a verification of this statement.

a) The series has all positive terms. Therefore if the "infinite tail" $\sum_{j=101}^{\infty} \frac{1}{j^3+2j^2+j}$ converges and has sum less than .001, the omitted terms after the first 100 of the whole series won't matter to 3 decimal places.

b) Overestimate the infinite tail $\sum_{j=101}^{\infty} \frac{1}{j^3+2j^2+j}$ by the infinite tail of a simpler series. Then compare the infinite tail of this simpler series to a simple improper integral. Use a diagram to help explain the comparison. Compute the improper integral.