Problem statement A computer reports the following information:

$$\sum_{j=1}^{10} \frac{30^j}{(j!)^2} \approx 6963.86479\,; \quad \sum_{j=1}^{15} \frac{30^j}{(j!)^2} \approx 6977.78140\,; \quad \sum_{j=1}^{20} \frac{30^j}{(j!)^2} \approx 6977.78249\,.$$

This suggests that $\sum_{j=1}^{\infty} \frac{30^j}{(j!)^2}$ converges and that its value (to at least 2 decimal places) is 6977.78. Explain the details in the following outline of a verification of this statement. In what follows, $a_j = \frac{30^j}{(j!)^2}$.

a) The series has all positive terms. Therefore if the infinite tail $\sum_{j=16}^{\infty} a_j$ converges and has sum less than .002, the omitted terms after the first 15 of the whole series won't matter to 2 decimal places.

b) If j is a positive integer, simplify the algebraic expression $\frac{a_{j+1}}{a_j}$. Use this to show that if $j \geq 16$, then $\frac{a_{j+1}}{a_j} < 0.11$. (Show all steps. You'll need a calculator!)

c) It is true that $a_{16} \approx .000983$. Use this fact and what was done in b) to compare $\sum_{j=16}^{\infty} a_j$ to a geometric series, each of whose terms is individually larger than this series. Find the sum of the geometric series, which should be less than .002, so that the omitted infinite tail of the original series is small enough. (Show all steps. You'll need a calculator!)