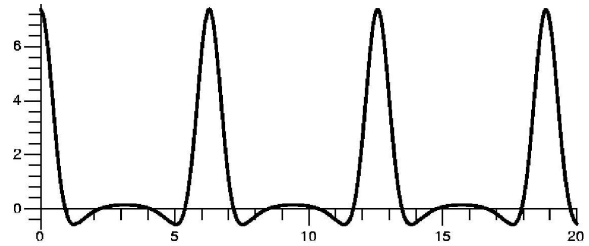


Problem statement Define $f(x)$ with the sum

$f(x) = \sum_{n=0}^{\infty} \frac{2^n \cos(nx)}{n!}$. This series is not a power series.

To the right is a graph of the partial sum

$s_{100}(x) = \sum_{n=0}^{100} \frac{2^n \cos(nx)}{n!}$ for $0 \leq x \leq 20$.



a) Verify that the series defining $f(x)$ converges for all x .

b) Is the apparent periodicity of the function $f(x)$ actually correct? If yes, explain why. Your explanation should include use of the term “periodic function” and explain why the defining condition is or is not satisfied.

c) Verify that the actual graph of the function is always within .01 of the graph shown. That is, if x is any real number, then $|f(x) - s_{100}(x)| < .01$.

Possibly useful numbers $2^{100} \approx 1.27 \cdot 10^{30}$ and $2^{101} \approx 1.54 \cdot 10^{30}$. Also, $100! \approx 9.33 \cdot 10^{157}$ and $101! \approx 9.43 \cdot 10^{159}$.