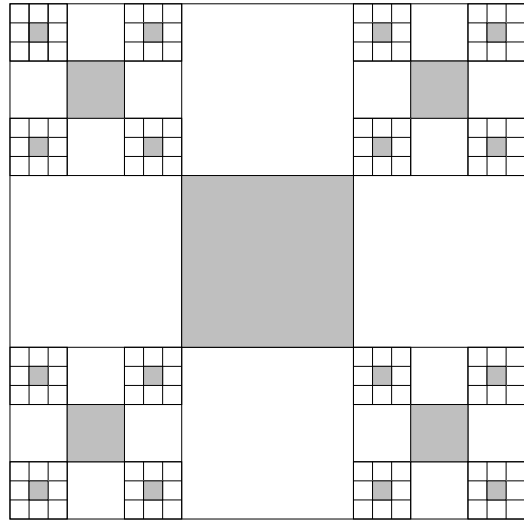


**Problem statement** A  $1 \times 1$  square is “dissected” by three equally spaced horizontal lines and by three equally spaced vertical lines. The central square is shaded. Then the bordering Northeast, Northwest, Southeast, and Southwest squares are similarly dissected, with the central square shaded. Each of *those* dissected squares has a similar process done to their borders, etc. The diagram to the right shows this only for the first three steps but it is supposed to continue indefinitely.



a) How many new shaded squares are introduced at the  $n^{\text{th}}$  step? (There is one shaded square at the first step.) What is the side length of the squares which are introduced at the  $n^{\text{th}}$  step?

b) What is the sum, as  $n$  goes from 1 to  $\infty$ , of the shaded area (all the shaded squares)? What is the sum, as  $n$  goes from 1 to  $\infty$ , of the perimeters of all the shaded squares?