**Problem statement** a) Suppose $f(x) = x^3$ and $g(x) = 4\cos(7x + 5) + 8\sin(x^2 - 9) + 6$. Find specific numbers $A$ and $B$ so that all values of $g$ are between $A$ and $B$ (that is, $A \leq g(x) \leq B$ for all $x$). The values of $A$ and $B$ don't have to be precise! Find a value of $x$ (call it $x_A$) so that $f(x_A) < A$ and another value of $x$ (call it $x_B$) so that $f(x_B) > B$.

b) Make a rough sketch on the same graph of $y = f(x)$ and $y = g(x)$ and $y = A$ and $y = B$ for $x$ between $x_A$ and $x_B$.

c) Find one root of $f(x) = g(x)$ approximately.

d) Explain why the following result is correct: **if** $F$ and $G$ are continuous functions defined on all real numbers and **if** $\lim_{x \to +\infty} F(x) = +\infty$ and $\lim_{x \to -\infty} F(x) = -\infty$ and **if** $G$ is bounded (this means there are numbers $A$ and $B$ so that $A \leq G(x) \leq B$ for all $x$) then the equation $F(x) = G(x)$ **must** have at least one root. (Look up the Intermediate Value Theorem.)