Problem statement About a decade ago three centuries of effort by mathematicians culminated in a proof that there were no solutions to the Fermat equation

$$a^n + b^n = c^n$$

if a, b, c, and n are positive integers, with $n > 2$. There are, of course, solutions when $n = 2$: for example, $3^2 + 4^2 = 5^2$.

a) Does the equation
$$4^x + 5^x = 6^x$$

have any solution? (The word "integer" does not appear in the preceding sentence!) If there is a solution, find an approximate value of this solution with accuracy $\pm.05$. If there is no solution, explain why.

b) Suppose a, b, and c are positive real numbers. Explore whether the equation

$$a^x + b^x = c^x$$

must have a solution. This is a "free form" question: try to answer it as well as you can. You are not asked to provide a "formula" for x. You are asked to find conditions which will guarantee that such an x either does or does not exist.