Problem statement A computer program reports the following:

$$\int_0^1 \frac{x}{x+1}\,dx = 1 - \ln 2\,; \qquad \int_0^\infty \frac{t}{(2t+1)(t+1)^2}\,dt = 1 - \ln 2\,.$$

Verify that the two integrals are equal. Notice that you are *not* asked to evaluate these definite integrals, only to explain why the values are equal.

Hint Find the antiderivatives and compute both integrals: a very direct method.

(Hint)^2 Change one integral into the other: x goes from 0 to 1 and t, from 0 to ∞ – everything involved is a rational function, so make the change from x to t with a simple rational function. After you find a suitable change of variables, how does dx change to dt?