

**Problem statement** a) Prove: if  $0 \leq x \leq 10$ , then  $0 \leq \sqrt{x+1} \leq 10$ .

b) Prove: if  $0 \leq u \leq v \leq 10$ , then  $0 \leq \sqrt{u+1} \leq \sqrt{v+1} \leq 10$ .

c) Consider the following recursively defined sequence:

$$a_1 = 0.3; a_{n+1} = \sqrt{a_n + 1} \text{ for } n > 1.$$

and compute its first five terms. Prove that this sequence converges. You should mention a specific **THEOREM** from section 10.1 which, together with the algebraic results of parts a) and b), will guarantee convergence.

d) What is the exact limit of the sequence defined in c)? (Square the recursive equation and take limits using some limit theorems.)

**Note** Many sequences encountered in “real life” are recursively defined, and the method in d) is often used. On many calculators, you can define the function  $\sqrt{x+1}$  and then use the **ANS** key repeatedly to calculate many terms of the sequence easily.