Problem statement a) The improper integral converges: $\int_0^\infty xe^{-x^2}\,dx$. What is its value?

b) The value of $\int_0^\infty e^{-x^2}\,dx$, another convergent improper integral, is $\frac{\sqrt{\pi}}{2}$. This amazing fact is easiest to explain with some of the tools in third semester calculus. Improper integrals involving polynomials and e^{-x^2} often arise in statistics and therefore in analysis of experiments. Use integration by parts to get a formula relating $\int_0^\infty x^n e^{-x^2}\,dx$ and $\int_0^\infty x^{n-2}e^{-x^2}\,dx$, where n is a positive integer bigger than 2. (The parts to take are slightly tricky.)

c) Now find the values of

i) $\int_0^\infty x^2 e^{-x^2}\,dx$ ii) $\int_0^\infty x^3 e^{-x^2}\,dx$ iii) $\int_0^\infty x^4 e^{-x^2}\,dx$

You will need the reduction formula in b) and the two initial values found in a) and b).