

Problem statement a) For x near 0, $\sin x$ is well-approximated by its tangent line at $x = 0$. What is this tangent line?

b) Approximation over an interval is preferred over approximation near a point for many purposes. One criterion for assessing the accuracy of such an approximation is *mean-square error*. The mean-square error between a straight line $y = Ax$ going through the origin and the function $\sin x$ over the interval $[0, 1]$ is given by the definite integral $\int_0^1 (\sin x - Ax)^2 dx$. Find the A which minimizes this integral.

Hint Expand the integrand, compute the integral, and find the A minimizing the result.

c) Sketch $\sin x$ and the straight lines found in a) and b) on the unit interval $[0, 1]$.