

Problem statement Investigate the family of curves defined by the parametric equations $x = t^2$, $y = t^3 - ct$, where c is a positive constant.

a) Graph the curves for $c = \frac{1}{4}$, $c = 1$ and $c = 4$. What features do all the curves have in common? (You may need to adjust the graphing window as you change c .) How does the shape change as c increases? Find the x - and y -coordinates of all points where the tangent line is horizontal or vertical.

b) Verify that the point $(c, 0)$ is on the curve for any $c > 0$. How many tangent lines does the curve have at the point $(c, 0)$? What are their slopes?

Check your answer numerically (for $c = \frac{1}{4}$, $c = 1$ and $c = 4$) by drawing the tangent lines on your graphing calculator.

c) Consider the curve corresponding to $c = \frac{1}{3}$. Part of this curve is a loop. Find the length of that loop.