Problem statement The convergent improper integral $\int_0^\infty e^{-x^2}\,dx$ is important in probability and statistics. An amazing result of multivariable calculus shows that the value is exactly $\frac{\sqrt{\pi}}{2}$. This problem shows how to get a numerical approximation of the integral's value.

a) Suppose $a \geq 1$. Use a substitution to calculate $\int_a^\infty xe^{-x^2}\,dx$. Now use this result to show that $\int_a^\infty e^{-x^2}\,dx \leq \frac{1}{2}e^{-a^2}$.

b) Calculate an approximate value for $\int_0^\infty e^{-x^2}\,dx$ as follows: find a value of a so that $\frac{1}{2}e^{-a^2} < 10^{-5}$. Now calculate $\int_0^a e^{-x^2}\,dx$ approximately using the numerical integration capability of your calculator.

c) Discuss all the sources of error in your calculation. Also discuss whether your answer is consistent with the claimed exact value of the integral in a).

d) Using your answer to b) calculate a numerical value for $\int_0^\infty x^2e^{-x^2}\,dx$.

Hint Write the integrand as $x(xe^{-x^2})$ and use integration by parts.