Problem statement A computer reports the following information:

$$\sum_{j=1}^{10} \frac{30^j}{(j!)^2} \approx 6963.86479; \quad \sum_{j=1}^{15} \frac{30^j}{(j!)^2} \approx 6977.78140; \quad \sum_{j=1}^{20} \frac{30^j}{(j!)^2} \approx 6977.78249.$$

This suggests that $\sum_{j=1}^{\infty} \frac{30^j}{(j!)^2}$ converges and that its sum (to 2 decimal places) is 6977.78. Explain the details in the following outline of a verification of this statement.

a) The series has all positive terms. Therefore if the "infinite tail" $\sum_{j=16}^{\infty} \frac{30^j}{(j!)^2}$ converges and has sum less than .002, the omitted terms after the first 15 of the whole series won't matter to 2 decimal places.

b) If $a_j = \frac{30^j}{(j!)^2}$, simplify the algebraic expression $\frac{a_{j+1}}{a_j}$. Use this to show that if $j \geq 16$, then $\frac{a_{j+1}}{a_j} < 0.11$. (You'll need a calculator!)

c) Assume (this is true!) that $a_{16} < .00099$. Use this fact and what was done in c) to compare $\sum_{j=16}^{\infty} a_j$ to a geometric series, each of whose terms is individually larger than this series. Find the sum of the geometric series, which should be less than .002. (Show all steps. You'll need a calculator!)