

**Problem statement** If a function  $f$  is continuous on the interval  $[a, b)$  and if  $\lim_{x \rightarrow b^-} f(x) = +\infty$  or  $-\infty$  then we say “ $f$  explodes at  $b$ .”

a) Consider the following functions on an interval  $[0, b)$  with  $b > 0$ . For each function, find  $b$  so that the function explodes at  $b$ . Use your calculator to show graphically what occurs.

$$\frac{x^2 + 1}{x - 1}; \quad \frac{\cos x}{x - 2}; \quad \frac{x - 2}{\cos x}.$$

b) Write all solutions to the differential equation  $y' = y^2$  subject to the initial condition  $y(0) = y_0$ . Can you find a solution that explodes at 10? And another one that explodes at 5? Can you find a solution that explodes at  $x_0$  when  $x_0 > 0$ ? How does the initial condition at  $x = 0$  connect with a specified explosion at  $x_0$ ? Graph one exploding solution.