Problem statement Define the function f by $f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!} = 1 - x^2 + \frac{x^4}{2} - \ldots$ (remember that $0! = 1$).

a) Determine the interval of convergence; this is the domain of f.

b) Write out several terms of the series and verify that $f'(x) = -2xf(x)$ for all x in the interior of the interval of convergence.

c) Show that $y = f(x)$ is a solution of the initial value problem $y' = -2xy, \quad y(0) = 1$.

d) Solve this initial value problem and get a formula for $f(x)$ in terms of functions found on your calculator.

e) Use the formula discovered for $f(x)$ and graph both f and the partial sum $s_6(x) = 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6}$ in a window where $0 \leq x \leq 1.2$. Then use the alternating series error formula to obtain an upper bound for the error in the approximation $f(x) \approx s_6(x)$ when $0 \leq x \leq 1.2$. Your answer should be a single number that applies to all x values in the range $0 \leq x \leq 1.2$, and it should be consistent with the graphs you have drawn.