Problem statement Under the hypotheses of the integral test, if $a_n = f(n)$ and if $s_n = a_1 + a_2 + \cdots + a_n = \sum_{j=1}^{n} a_j$, then $\int_1^n f(x)\,dx \le s_n \le a_1 + \int_1^n f(x)\,dx$ for each positive integer n.

For the harmonic series $\sum_{j=1}^{\infty} \frac{1}{j}$, this implies $\ln n \le 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \le 1 + \ln n$ for each positive integer n.

a) Find the analogous inequalities for the series $\sum_{j=1}^{\infty} \frac{1}{\sqrt{j}}$ and for the series $\sum_{j=2}^{\infty} \frac{1}{j \ln j}$.

b) Estimate the sum of the first 10^{10} terms of the series, in each of the three cases. Then estimate the sum of the first 10^{100} terms.

c) Of the three series, which diverges the fastest? the slowest?